OR

(Classical Dynamics)

[CBCS]

Full Marks : 60 Time : Three Hours

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

Group - A

Attempt any *ten* : 2×10=20

1. A rod of length L_0 moves with speed v along horizontal direction. The rod makes an angle θ_0 with respect to the x' axis. Determine the angle θ the rod makes with the x axis. 2

2. Consider two masses m_1 and m_2 moving in three dimensions, which are attracted to each other gravitationally, and are also acted on by a uniform gravitational field with acceleration strength g in the $-\hat{z}$ direction. Write down the Lagrangian. (Hint : You may use the concept of reduced mass and polar coordinates.) 2

3. What is the physical significance of Laplace-Runge-Lenz vector? 2

4. Two events separated by time-like interval cannot be causally related. *True* or *False*? 2

5. Show that, $E^2 = p^2c^2 + m^2c^4$ (symbols hold their usual meaning). 2

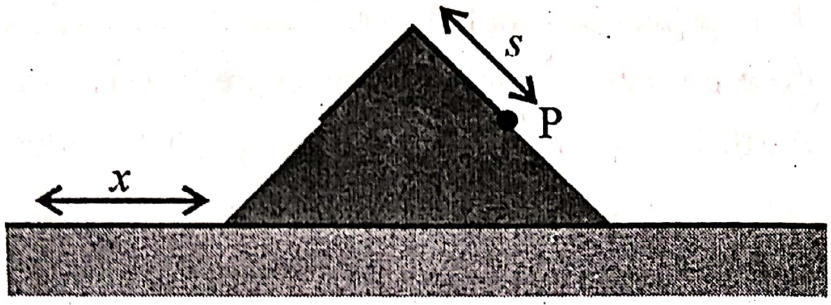
6. Two powerless rockets are heading towards each other on a collision course. As measured by Liz, a stationary Earth observer, Rocket 1 has speed 0.800c, Rocket 2 has speed 0.600c, both rockets are 50.0 m in length, and they are initially 2.52 Tm apart. ($1Tm = 10^{12}m$). What is the length of each rocket as observed by a stationary observer in the other rocket? 2

7. Atwood's machine consists of two blocks of mass m_1 and m_2 attached by an inextensible cord which suspends them from a pulley of moment of inertia I with frictionless bearings. Write down the Lagrangian of the system. 2

8. The production of Higgs particles at the LHC was achieved through the collision of two collinear beams of protons. We consider the collisions in the rest frame K of the LHC. We denote by $M=125\ GeV/C^2$ the mass of the Higgs particle, and by $m = 938\ MeV/c^2$ the mass of a proton. Suppose two protons move along the x-axis with equal but opposite velocities to produce a Higgs particle at rest upon collision. Find the required energy and velocity of the protons. 2

9. A uniform chain of length l and mass m lies on top of a triangle block of mass 2m with two equal sides of

length ≥ 1 and the top corner forming a right angle. The block is free to slide on a frictionless horizontal plane.



At the right end P of the chain, a mass m is attached. The system is subjected to the gravity force and we ignore friction. At time $t = 0$ it is at rest with a length $s(0) = 1/2$ of chain lying from the right side of the block. Write down the Lagrangian of the system. 2

10. Two clocks are positioned at the ends of a train of length L (as measured in its own frame). They are synchronized in the train frame. The train travels past you at speed v. It turns out that if you observe the clocks at simultaneous times in your frame, you will see the rear clock showing a higher reading than the front clock. By how much? 2

11. The stream of water flowing at high speed from a garden hose pipe tends to spread like a fountain when held vertically up, but tends to narrow down when held vertically down. *True* or *False*? 2

12. What can you do to convert a laminar flow into turbulent flow? 2

13. When a particle is constrained to move on the surface

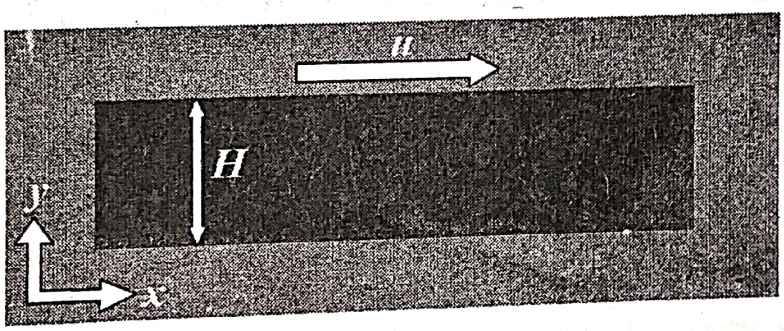
of a cylinder of radius R, the Lagrangian is given in cylindrical coordinates by :

$$L=\frac{1}{2}m\left(R^2\dot{\varphi}^2+\dot{z}^2\right)-\frac{1}{2}k\left(z^2+R^2\right)-mgz$$

Obtain the corresponding Hamiltonian. 2

14. Describe the physical origin of all the terms in Navier Stokes equation. 2

15. Set up the Navier Stokes equation for the Couette Flow in the steady state and fully developed flow (diagram below) : 2



Group - B

Attempt any ***four*** : 5×4=20

1. A train with proper length L moves at speed 5c/13 with respect to the ground. A ball is thrown from the back of the train to the front. The speed of the ball with respect to the train is c/3. As viewed by someone on the ground, how much time does the ball spend in the air, and how far does it travel? 5

2. Use a Minkowski diagram where the axes in frame S are orthogonal to solve the following problems :

   (a) The relative speed of S' and S is v (along the x

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direction). A meter stick lies along the x' axis and is at rest in S'. If S measures its length, what is the result?

(b) Now let the meter stick lie along the x axis and be at rest in S. If S' measures its length, what is the result? 3+2

3. Consider a mass "m" on the end of a massless rigid rod of length "L", the other end of which is free to rotate about a fixed point. This is a spherical pendulum. Find the Lagrangian and the equations of motion. 5

4. Find a differential equation for $\theta(\varphi)$ for the shortest path on the surface of a sphere between two arbitrary points on that surface, by minimizing the length of the path, assuming it to be monotone in φ. 5

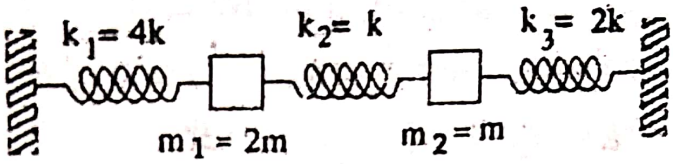
5. A particle of mass m moves in one dimension such that it has the Lagrangian as follows :

$$L = \frac{m^2\dot{x}^4}{12} + m\dot{x}^2 f(x) - V(x)$$

Here $f(x)$ is some differentiable function of x. Find the equation of motion and describe the physical nature of the system based on this equation. 5

6. (a) The mass flow rate through a cylindrical pipe of cross-sectional area $A = 0.5$ m^2 is $\dot{m} = 1500$ kg/s. What is the pressure difference over a distance $L = 10$ m? Viscosity of water is 1.002 milli Pa-s. 3

(b) Write down the Lagrangian for the following system : 2



Group - C

Attempt any *two* : 10×2=20

1. Consider the Navier equation ignoring the volume force, and show that :

   (a) A uniform elastic material can support longitudinal waves. At what speed do they travel? 3

   (b) A uniform elastic material can support transverse waves. At what speed do they travel? 3

   (c) Granite has a density of 2700 kg/m^3, a bulk modulus of 4×10^{10} N/m^2 and a shear modulus of 2.5×10^{10} N/m^2. If a short spike of transverse oscillations arrives 25 seconds after a similar burst of longitudinal oscillations, how far away was the explosion that caused these waves? 4

2. A particle of mass m_1 moves in two dimensions on a frictionless horizontal table with a tiny hole in it. An inextensible massless string attached to m_1 goes through the hole and is connected to another particle of mass m_2, which moves vertically only. Give a full set of

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generalized unconstrained coordinates and write the Lagrangian in terms of these. Assume the string always remains taut and there is no friction of the string sliding at the hole. Are there ignorable coordinates? Reduce the problem to a single second order differential equation. Show this is equivalent to single particle motion in one dimension with a potential $V(r)$, and find $V(r)$.

3+2+3+2

3. Find out the eigenfrequencies of oscillation of a linear triatomic molecule. Discuss the physical implications of zero eigen frequency. 6+4

4. An object moves at speed $v_1/c = \beta_1$ with respect to S_1, which moves at speed β_2 with respect to S_2, which moves at speed β_3 with respect to S_3 and so on, until finally S_{N-1} moves at speed β_N with respect to S_N (See Figure). Show by induction that :

$$\beta_N = \frac{P_N^+ - P_N^-}{P_N^+ + P_N^-} \text{ where } P_N^+ = \Pi_{i=1}^N (1+\beta_i) \text{ and}$$

$$P_N^- = \Pi_{i=1}^N (1-\beta_i)$$

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